$$(\cos x)^{2m} = \left(\frac{e^{ix}+e^{-ix}}{2}\right)^{2m}$$ užijeme binom. větu

$$= \sum_{k=0}^{2m} \binom{2m}{k} \frac{1}{2^{2m}} \left(e^{ix}\right)^{2m-k} \left(e^{-ix}\right)^{k}$$

$$= \sum_{k=0}^{2m} \binom{2m}{k} \frac{1}{2^{2m}} e^{ix(2m-2k)}$$

$$= \sum_{k=0}^{m-1} \binom{2m}{k} \frac{2}{2^{2m}} \frac{e^{ix(2m-2k)} + e^{-ix(2m-2k)}}{2}$$

$$+ \binom{2m}{m} \frac{1}{2^{2m}} \cdot e^{0}$$

$= \cos(2m-2k)x$

Takže

$$\boxed{(\cos x)^{2m} = \frac{\binom{2m}{m}}{2^{2m}} + \sum_{n=1}^{m} \frac{2\binom{2m}{m-n}}{2^{2m}} \cos(2nx)}$$

$$(\cos x)^{2m+1} = \left(\frac{e^{ix}+e^{-ix}}{2}\right)^{2m+1}$$ užijeme binom. větu

$$= \sum_{k=0}^{2m+1} \binom{2m+1}{k} \frac{1}{2^{2m+1}} \left(e^{ix}\right)^{2m-k+1} \left(e^{-ix}\right)^{k}$$

$$= \sum_{k=0}^{2m+1} \binom{2m+1}{k} \frac{1}{2^{2m+1}} e^{ix(2m+1-2k)}$$

$$= \sum_{k=0}^{m} \binom{2m+1}{k} \frac{2}{2^{2m}} \frac{e^{ix(2m+1-2k)} + e^{-ix(2m+1-2k)}}{2}$$

$$= \cos(2m+1-2k)x$$

Takže

$$\boxed{(\cos x)^{2m+1} = \sum_{n=0}^{m} \frac{2\binom{2m+1}{m-n}}{2^{2m}} \cos(2n+1)x}$$